V-Escuela del Doctorado en Matemática de Valparaíso

Universidad Técnica Federico Santa María - Universidad de Valparaíso - Pontificia Universidad Católica de Valparaíso

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Advanced Discretization: Theory, Applications and Numerical Implementation

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Abstract

In this short course we aim to cover fundamental aspects of advanced discretization algorithms for partial differential equations (PDEs). We first give an overview of the mathematical theory behind so-called Petrov–Galerkin methods, focussing on solvability of equations, stability of discretizations and analysis of convergence. We illustrate the abstract framework by applying it to various PDE examples. Subsequently, we deliver certain tools on how to code the method, showing that the theory matches with numerical experiments. This short course will be taught in the English language. Students will have an opportunity to test and develop code using MATLAB (or other programming language), and they are encouraged to bring their own laptop.

Contents by class

1. Motivational examples: linear PDEs. Solvability of linear equations in Banach spaces: the Banach closed-range Theorem and bounded-below operators.

2. Application to linear equations in weak forms: the inf–sup condition and the Lax–Milgram Theorem. Examples: elliptic, hyperbolic, parabolic, mixed problems.

3. Approximation Theory: Best and quasi-best approximation, Petrov–Galerkin discretizations, error estimation, Babuška's Theory and Céa's Lemma.

4. Numerical implementation: Galerkin finite element method for elliptic PDEs, Petrov-Galerkin for hyperbolic PDEs.

5. Mixed problems: Compatible discretization, Babuška's–Brezzi Theory, Mixed Galerkin FEM, numerical implementation.

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